EXPERIMENTAL STUDY OF THE TRANSIENT HEAT TRANSFER BETWEEN METAL BALLS AND A STREAM OF LIQUID AT CONSTANT TEMPERATURE

A. V. Lykov, B. M. Smol'skii, and L. A. Sergeeva

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The article describes a method of determining the transient heat-transfer coefficient and a study of the transient heat transfer between metal balls and a stream of liquid. It is shown here how the heat flux and the heat-transfer coefficient depend on a number of parameters. The relation is then generalized for a transient heat flux at a surface.

Inasmuch as most of the work done on the subject has dealt with steady-state heat-transfer problems, the intensity of heat transfer was considered to depend on the hydrodynamics of the stream around the immersed body, i.e., on the value of coefficient α, which is a function of the flow velocity, the properties of the fluid, and the characteristic dimensions of the body only.

In many processes accompanied by a heat transfer between a solid body and a fluid, however, either the temperature of the body surface or the heat flux at that surface changes with time. In recent years there has been felt a growing interest in the study of transient heat-transfer processes, as such processes are often encountered in modern technology [1, 2].

Experimental data pertaining to the intensity of transient heat transfer between a ball and a stream of liquid are missing in the published literature. We have seen the results of studies [3, 4, 5] concerning the heat transfer between balls and an air stream under transient conditions. In these studies, however, there appear certain contradictions. Thus, for example, in [3, 5] it was discovered that the heat-transfer coefficient had remained constant even during the first few seconds of cooling. In [4], on the other hand, the heat-transfer coefficient was 2.5 times greater during the first few seconds than after steady-state had been reached. Such divergent results were obtained even though the air flow parameters in those investigations were nearly the same. All balls used in the experiments met the requirement that $Bi < 0.1$ [translator's note: for definition of Bi see notation at the end of this article] and that the initial temperature difference be of the order of several tens of degrees. The same method of determining the heat-transfer coefficient was used in every case.

The object of our study is to establish, using as a test case the heat transfer between balls and a water stream, whether there is a difference between the steady-state and the transient heat-transfer intensity, and also to analyze the effect of basic process parameters on the transient heat-transfer intensity.

The methods and the results of transient heat-transfer coefficient measurements will be presented here. The conclusions based on a comparison between the transient and the steady-state heat-transfer intensities will be published later.

The transient heat flux and heat-transfer coefficient were determined in our study by three different methods.

1. The authors developed a method of determining the conditions of heat transfer at the surface of thick-walled hollow balls (with large values of the Bi parameter). To this end, they considered the problem of a hollow sphere being heated while the temperatures of its outer and its inner surface are known functions

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of time. The heat-transfer equation for the particular case is:

$$\frac{\partial \vartheta}{\partial \tau} = a \frac{\partial^2 \vartheta}{\partial r^2} + \frac{2}{r} \frac{\partial \vartheta}{\partial r}. \qquad (1)$$

On the basis of experience, one may approximate the temperature change at a surface of the sphere as the following function of time:

$$\vartheta = A[1 - \exp(-k\tau)]. \qquad (2)$$

In this way, we have used

$$\begin{aligned} r = r_1 \quad & \vartheta(r_1, \tau) = A_1[1 - \exp(-k_1\tau)], \\ r = r_2 \quad & \vartheta(r_2, \tau) = A_2[1 - \exp(-k_2\tau)] \end{aligned} \qquad (3)$$

as the boundary conditions and $\tau = 0$, $\vartheta(r, 0) = 0$ as the initial condition in the analysis.

The solution of the given problem will be

$$\begin{aligned} \vartheta(r, \tau) = {} & A_1 \frac{r_1(r_2 - r)}{r(r_2 - r_1)} + A_2 \frac{r_2(r - r_1)}{r(r_2 - r_1)} - A_1 \frac{r_1}{r} \frac{\sin\left[\sqrt{\frac{k_1}{a}}(r_2 - r)\right]}{\sin\left[\sqrt{\frac{k_1}{a}}(r_2 - r_1)\right]} \\ & \times \exp(-k_1\tau) - A_2 \frac{r_2}{r} \frac{\sin\left[\sqrt{\frac{k_2}{a}}(r - r_1)\right]}{\sin\left[\sqrt{\frac{k_2}{a}}(r_2 - r_1)\right]} \exp(-k_2\tau) \\ & + 2A_1 \frac{r_1}{r} \sum_{n=1}^{\infty} (-1)^n \frac{1}{n\pi} \sin\left[n\pi \frac{r_2 - r}{r_2 - r_1}\right] \exp\left[-\frac{an^2\pi^2}{(r_2 - r_1)^2}\tau\right] \\ & + 2A_2 \frac{r_2}{r} \sum_{n=1}^{\infty} (-1)^n \frac{1}{n\pi} \sin\left[n\pi \frac{r - r_1}{r_2 - r_1}\right] \exp\left[-\frac{an^2\pi^2}{(r_2 - r_1)^2}\tau\right] \\ & - 2A_1 \frac{r_1}{r} \sum_{n=1}^{\infty} (-1)^n \frac{an\pi}{an^2\pi^2 - k_1(r_2 - r_1)^2} \sin\left[n\pi \frac{r_2 - r}{r_2 - r_1}\right] \\ & \times \exp\left[-\frac{an^2\pi^2}{(r_2 - r_1)^2}\tau\right] - 2A_2 \frac{r_2}{r} \sum_{n=1}^{\infty} (-1)^n \frac{an\pi}{an^2\pi^2 - k_2(r_2 - r_1)^2} \\ & \times \sin\left[n\pi \frac{r - r_1}{r_2 - r_1}\right] \exp\left[-\frac{an^2\pi^2}{(r_2 - r_1)^2}\tau\right]. \end{aligned} \qquad (4)$$

Next, the contribution of each series term to the total sum was estimated. With the particular ball materials and dimensions selected for this study, it turned out that some of the series terms contribute approximately 1% to the temperature change (with $n = 1$, still less when $n > 1$) and these terms were disregarded in subsequent calculations. An analogous estimate was made for the heat flux equation resulting from differentiation of Eq. (4) and insertion into the Fourier equation. The heat flux due to several terms in the series does not exceed 4% at worst. This made it possible to simplify the temperature and the heat flux equations. The heat flux at the surface during the heat transfer between a hollow sphere and a moving medium becomes then

$$\begin{aligned} q = -\lambda \Bigg\{ & \frac{A_1 r_1}{r_2(r_2 - r_1)} - \frac{A_2 r_1}{r_2(r_2 - r_1)} - \frac{A_1 r_1 \sqrt{\frac{k_1}{a}}}{r_2 \sin\left[\sqrt{\frac{k_1}{a}}(r_2 - r_1)\right]} \exp(-k_1\tau) \\ & + A_2 \sqrt{\frac{k_2}{a}} \operatorname{ctg}\left[\sqrt{\frac{k_2}{a}}(r_2 - r_1)\right] \exp(-k_2\tau) - \frac{A_2}{r_2} \exp(-k_2\tau) \Bigg\}. \end{aligned} \qquad (5)$$

2. For thin-walled balls with a high thermal conductivity the authors have, in addition, used the method of determining the boundary conditions on the basis of the smallness criterion for the parameter Bi. In this case the quantity of heat supplied to the body during a time interval $d\tau$ is equal to its thermal capacity multiplied by the temperature change dt. Thus, we have

$$Qd\tau = G_{B}c_{p_{B}}dt = \alpha(\tau)(t_{liq} - t)Fd\tau. \tag{6}$$

After necessary transformations, the following expression is obtained for the heat-transfer coefficient when the wall of a hollow ball is thin:

$$\alpha(\tau) = -c\rho\delta\frac{d\ln\vartheta}{d\tau}. \tag{7}$$

3. The third method used in this study was that of successive intervals. It applies to thin-walled balls as well. The authors of [7] give a solution to the homogeneous heat conduction equation applied to a plate for successive time intervals, where the final temperatures determined for any interval become the initial conditions for the next interval. The range of values Fo ≤ 0.5 [translator's note: for definition of Fo see notation at the end of this article] was considered for each time interval. Under this premise one may, with little error, break off the infinite sum of the series terms after the first few ones in the temperature equation. From the solution for the n-th time interval we obtained the following expression for the heat flux:

$$q_n = \frac{[t(x,\ \tau) - t_0]\frac{\lambda}{z} - \sum\limits_{i=1}^{i=n-1} q_i \mathrm{Fo}_i}{\mathrm{Fo}_n - \frac{1}{6} + \frac{x^2}{2z^2}}, \tag{8}$$

where z is the plate thickness.

For x = z, knowing the heat flux, one can find from Eq. (8) the surface temperature and then the heat-transfer coefficient α. The procedure for determining the heat flux at the surface of a solid sphere is the same as for the case of a hollow sphere. For a solid spherical body the equation becomes

$$q = -\lambda\left\{-A_2\sqrt{\frac{k_2}{a}}\,\mathrm{ctg}\left[\sqrt{\frac{k_2}{a}}\,r_2\right]\exp(-k_2\tau) + \frac{A_2}{r_2}\exp(-k_2\tau)\right\}. \tag{9}$$

With the aid of Eq. (9) we calculated the heat-transfer coefficient for one of the cases shown in [5]: for a lead ball with a diameter 29.9 mm, λ = 34.9 W/m · deg, a = 0.084 m^2/h, t_0 = 100°C, t_{liq} = 21.9°C. The expression for the excess temperature at the surface of this ball was found to be:

$$\vartheta = 78[1 - \exp(-0.011\tau)].$$

The surface temperature calculated from this formula differs from that determined experimentally by not more than 1.4%. The mean heat-transfer coefficient for a lead ball facing a stream with a velocity of w = 11.7 m/sec was found to be α = 81.2 W/m^2 · deg, which agrees with the data obtained in [3, 5]. In this way, our method of determining the conditions at the heat-transfer boundary between a spherical body and a surrounding medium was checked out indirectly.

Only in [7] was the transient heat transfer between a solid body and a water stream ever investigated experimentally. The superior feature of our study is that, while the transient heat transfer is investigated using balls, the need for thermal insulation of the test piece is obviated and corresponding heat losses do not have to be accounted for. In our study, unlike in [7], the objective was to compare the heat-transfer intensities under steady-state and transient conditions, and to determine how the relation between both cases depends on the basic parameters of the transient heat transfer. Besides, with such a design of the experiment, the specimen characteristics averaged over its surface are equal to their local values. The same applies also to the temperature of the liquid.

The test apparatus for studying the transient heat transfer with balls consisted of two thermostats, a model VSA-6M direct-current supply, a model N-700 oscillograph, a model PP-0.05 potentiometer, a Dewar flask, and a ball-cooling system (for steady-state heat-transfer tests).

The thermostats were used for establishing the initial temperature, for heating the specimens, and for tracking the steady-state heat transfer. The light-beam oscillograph was used for recording temperature changes with time at a series of points on a specimen. The oscillograph timer could not be used at the given strip chart velocity. For this reason, a separate timer was connected to one of the oscillograph transducers. The heating of the water and the temperature distribution across the specimen wall during steady-state heat-transfer tests were measured with the potentiometer.

Metal balls with the same outside diameter of 100 mm were used as test specimens. With equal outside diameters one could compare the test results under the same external conditions which determine the heat-transfer coefficient. Four balls were made of copper. Three of them were hollow with $\delta = 5, 15, 25$ mm wall thicknesses and one was solid. Two hollow balls had a $\delta = 25$ mm wall thickness, one made of aluminum and one made of brass. The thermophysical characteristics of these materials taken from [8] include the variation of their thermal conductivities λ from 110 to 390 W/m·deg and of their $c\rho$ parameters from 2420 to 3470 kJ/m^3·deg.

A ball was held securely inside a thin-walled stainless-steel tube, which also served as a conduit for the thermocouple wires. The ball was insulated from the tube by a Teflon sleeve with low thermal conductivity.

A textolite cap was fastened to the tube for positioning a ball inside the thermostat horizontally. The distance from the cap to a ball was maintained the same for all specimens. In this way, the location of balls inside the thermostat was fixed.

Each ball consisted of two halves with threaded joints.

The Chromel–Copel thermocouples used in the experiment were capable, together with the most sensitive transducers of the N-700 oscillograph, of measuring the temperature of a ball wall within satisfactory accuracy. The error of the temperature measurements was within 0.5-2.0%.

The hot junctions of the thermocouples were secured in a wall at various distances from the center. The thermocouple wires were laid along isothermal surfaces. In some balls certain thermocouples, namely those at the inner surface, were patched on at various points to check out whether the balls were heating up uniformly. The cold junctions of the thermocouples were held inside the Dewar flask at the melting temperature of ice.

The lag of the measuring system (thermocouple-transducer) was also determined. It amounted to 0.3 sec for a temperature change within the test range (from 27 to 97°C). The error in temperature determinations due to a lag of this magnitude did not exceed 1%.

The test sequence in this experiment was as follows. A ball was thermostaticized at the temperature of 27°C, this being necessary for establishing the initial condition $t(r, 0) = t_0 = \text{const}$. Then the strip chart mechanism of the oscillograph as well as the timer were switched on, the latter marking 0.5 sec intervals on the chart. Now the ball was transferred to the 97° thermostat. The heating of the ball was recorded for 2 min. This test was repeated 3-8 times with each ball. The excellent reproducibility of the results is evidence that the conditions remained sufficiently unchanged from test to test (Fig. 1b).

As a first step, the temperature as a function of time $t(r, \tau) = f(\tau)$ was derived from the measured data and oscillograms (Fig. 1) for various points along the wall depth of a ball.

Prior to that, before analyzing the experimental results, the general character of the process was examined.

First Procedure. If it is assumed that the heat-transfer coefficient in a given process remains constant over the entire time interval (i.e., the process is quasi-stationary), then the following expression will apply to a solid sphere [9]:

$$\theta = \frac{t - t_0}{t_{\text{liq}} - t_0} = 1 - \sum_{n=1}^{\infty} \frac{2(\sin\mu_n - \mu_n \cos\mu_n)}{\mu_n - \sin\mu_n \cos\mu_n} \frac{R \sin\mu_n \frac{r}{r_2}}{r\mu_n} \exp(-\mu_n^2 \text{Fo}). \tag{10}$$

It follows from an analysis of Eq. (10) given in [9] that in our case it is sufficient to consider only the first term of the summation when determining the ratio of temperature changes θ.

If with the aid of Eq. (10) the expressions for the temperature at the same radius but at different instants of time are written down, then these expressions combined will yield

$$\mu = \sqrt{\frac{\lg[(t_{\text{liq}} - t_1)/(t_{\text{liq}} - t_2)]}{\lg e(\text{Fo}_1 - \text{Fo}_2)}}.$$

Substituting into this expression the measured temperatures, we will find μ, Bi, and from there the heat-transfer coefficient α.

Second Procedure. Here the heat-transfer coefficient α is also assumed constant. The solution for the ratio of temperature changes θ is written for the same instant of time but at two different radii (r' and r''):

$$\theta_1 = \frac{t_{liq} - t_{r'}}{t_{liq} - t_0} = B \frac{r_2 \sin\left(\mu_1 \frac{r'}{r_2}\right)}{r'\mu_1} \exp(-\mu_1^2 \mathrm{Fo}),$$
$$\theta_2 = \frac{t_{liq} - t_{r''}}{t_{liq} - t_0} = B \frac{r_2 \sin\left(\mu_1 \frac{r''}{r_2}\right)}{r''\mu_1} \exp(-\mu_1^2 \mathrm{Fo}). \quad (11)$$

Combining both expressions gives

$$\sin\left(\mu_1 \frac{r'}{r_2}\right) = \frac{r'}{r''} \sin\left(\mu_1 \frac{r''}{r_2}\right) \frac{t_{liq} - t_{r'}}{t_{liq} - t_{r''}}. \quad (12)$$

In this case the values of coefficient α are determined by solving Eq. (12) graphically. Then, using the tabulated data in [9], we find the value of Bi and the heat-transfer coefficient α.

Calculations by the first and by the second procedure have shown that in our case the heat-transfer coefficient appears to be a variable and time-dependent quantity. It varied by a factor of 1.8-3.2 when determined by the first procedure for different radii and by a factor of 3 when determined by the second procedure. The absolute values of the heat-transfer coefficient determined in this way could not be accepted as the true values of α, since the proposed method is adequate only for estimating the character of the heat-transfer process (quasi-stationary or transient).

In order to produce quantitative results, the relations $t(r, \tau) = f(\tau)$ found experimentally were further used for determining the form of the boundary functions. Their interrelation was established by way of solving the problem for the case of an arbitrarily varying heat-transfer coefficient (4). As is evident from Eq. (5), the heat flux density at the heat-transfer surface can only be determined if the function which describes how the surface temperature varies with time is known. These relations were established by the method of successive approximations using Eq. (4) and on the basis of temperature–time measurements. For this purpose, the experimental curves were approximated by equations of the form $t = A[1 - \exp(-k\tau)]$. The deviation of measured temperature curves from the calculated ones did not exceed 0.5%.

The surface temperature of the copper ball with the $\delta = 5$ mm wall thickness was determined also by the method of successive intervals (8), this method being fully applicable to thin-walled bodies.

As a check, a thermocouple was patched on into one of the balls (the copper ball with the $\delta = 15$ mm wall thickness) at a distance of 1 mm below the surface. With the high thermal conductivity of copper, as was to be expected, the readings of this thermocouple came very close to the calculated values of surface temperature (Fig. 1a).

After the temperature as a function of time has been determined for the outer and the inner surface of a hollow ball according to (5), the heat fluxes were also calculated (Fig. 2). As the diagram shows, a different heat flux enters specimens with different wall thicknesses δ at the same instant of time during the heat-transfer process. The magnitude of that heat flux does, evidently, depend also on the heat capacity of the wall. As the wall becomes thicker and the parameter $c\rho$ increases, the heat flux also increases. If the heat flux is plotted versus time on semilogarithmic paper, then the result will be a family of straight lines whose slopes are a function of the parameter $1/\delta c\rho$.

This has led to a generalized expression for the heat flux as a function of time, of the wall thickness, and of the parameter $c\rho$, when these quantities vary within earlier specified limits:

$$q_{\text{н}} = 1.34 \cdot 10^5 \exp\left[-\left(\frac{1.49}{\delta c\rho} + 0.02\right)\tau\right], \quad (13)$$

with τ in seconds.

The maximum error of this formula is 15%.

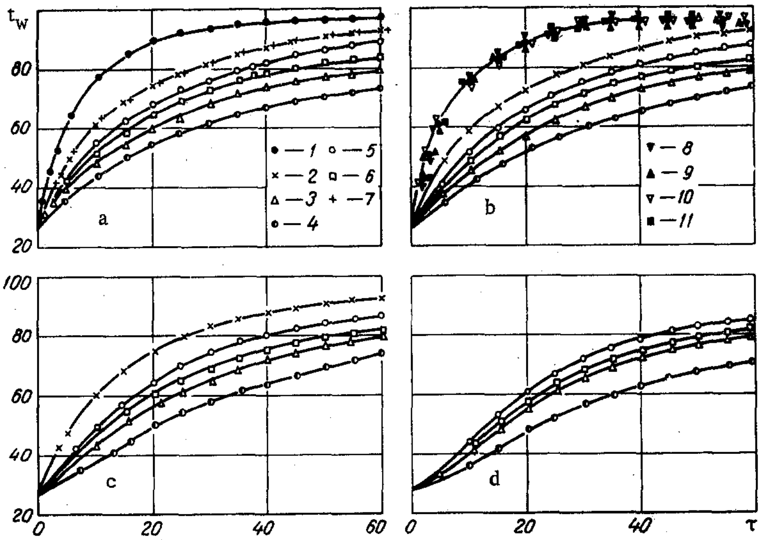

Fig. 1. Wall temperature t(°C) of specimens as a function of time τ (sec): a) $r = r_2$; b) $r = 0.9r_2$; c) $r = 0.7r_2$; d) $r = 0.5r_2$. 1) Copper $\delta = 0.005$ m; 2) copper, $\delta = 0.015$ m; 3) copper, $\delta = 0.025$ m; 4) copper, $\delta = 0.05$ m; 5) aluminum, $\delta = 0.025$ m; 6) brass, $\delta = 0.025$ m; 7) temperatures measured at a distance of 1 mm below the surface; 1, 8-11) wall temperatures measured at one point in different tests.

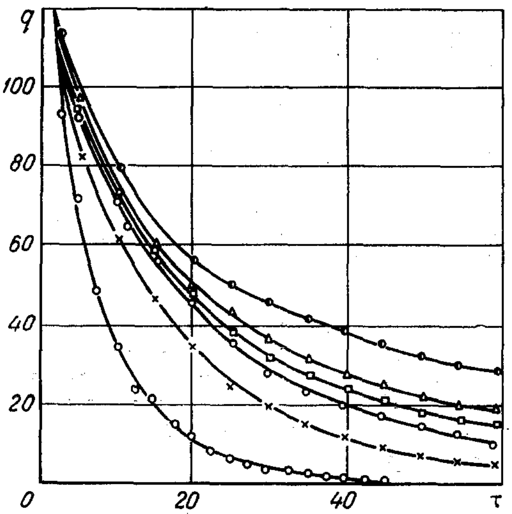

Fig. 2

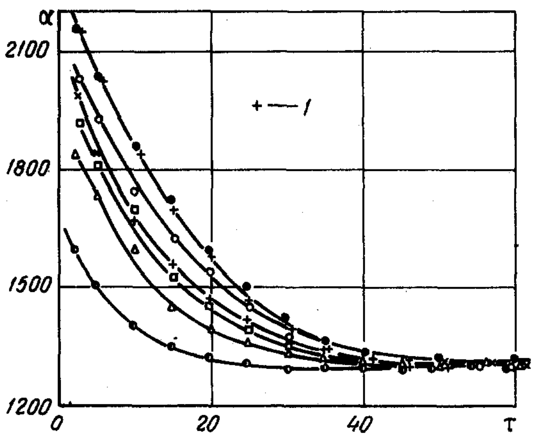

Fig. 3

Fig. 2. Heat flux q, $16^{-1} \cdot 10^{-3}$ W/m^2, as a function of time τ (sec). Legend the same as in Fig. 1.

Fig. 3. Heat-transfer coefficient α, $16^{-1} \cdot$ W/m$^2 \cdot$deg, as a function of time τ (sec): 1) calculated by Eq. (7). Other designations as in Fig. 1.

When solving the problem of transient heat transfer between a stream of fluid and a portion of a tube wall, the authors of [1] obtained the following special expression for the heat flux density at the surface:

$$q \cong A \exp\left[-\frac{B}{\delta c \rho}\tau\right]. \tag{14}$$

A comparison of (13) and (14) indicates that the nature of the relation between the heat flux and the basic parameters is analogous in both expressions. We thus experimentally confirmed the dependence of the transient heat flux on $c\rho$, δ, and τ which had been established theoretically solving the problem in accordance with [1]. It was not possible to make a more detailed comparison with the theoretical relation, since Eqs. (13) and (14) had been obtained in the course of analyzing only a few individual examples of transient heat transfer.

From the known values of heat flux (Fig. 2) and the surface temperature (Fig. 1a), the temperature of the liquid remaining constant, the heat-transfer coefficient as a function of time was determined for all balls (Fig. 3). For the copper ball with the 5 mm wall thickness the heat-transfer coefficient was determined from Eqs. (7) and (8). The results of determining α by the two methods are in fair agreement (Fig. 3).

It follows from Fig. 3 that under transient conditions the heat-transfer coefficient is a function of time and also depends on the wall thickness as well on the material characteristics of a given specimen.

The heat-transfer coefficients for all specimens gradually decreased with time, approaching a common constant value of $\alpha = 1510\ \mathrm{W/m^2 \cdot deg}$. The time interval within which the magnitude of the heat-transfer coefficient became stabilized was 30-50 sec, depending on the specimen. From that time on, the heat transfer became a quasi-stationary process. The heat-transfer coefficient for a hollow ball with a 5 mm wall thickness was 65% greater, and for a solid ball was 23% greater, during the first few seconds than during the quasi-stationary period.

NOTATION

t is the wall temperature of sphere (ball), °C;
t_0 is the initial wall temperature of sphere (ball), °C;
t_s is the temperature of ball surface, °C;
t_{liq} is the temperature of liquid, °C;
τ is the time;
r_1 is the inside radius of sphere (ball);
r_2 is the outside radius of sphere (ball);
r is the running radius;
d_{sph} is the diameter of sphere (ball);
δ is the thickness of wall;
a is the thermal diffusivity of a material, m^2/h;
λ is the conductivity of a ball material, W/m · deg;
q is the heat flux density, W/m^2;
c_{pw} is the heat capacity of liquid (water), kJ/kg · deg;
α is the heat-transfer coefficient, $W/m^2 \cdot deg$;
a_{liq} is the thermal diffusivity of liquid, m^2/h;
$c\rho$ is the volume heat capacity of ball materials, kJ/kg · deg;
$Bi = \alpha\delta/\lambda$;
$Fo = a\tau/\delta^2$.

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